PAPER SOLUTION

From Meerut

JEE MAIN

JAN	SHIFT
28	1st

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#Q. In an AP, $T_m = \frac{1}{25}$, $T_{25} = \frac{1}{20}$ and $20\sum_{r=1}^{25} T_r = 13$, then $5m\sum_{r=m}^{2m} T_r$ equals

$T_n = \frac{1}{m}$, $T_m = \frac{1}{n}$, $a = d = \frac{1}{mn}$

$m = 20$, $a = d = \frac{1}{25\times 20} = \frac{1}{500}$

$5\times 20 \sum_{r=20}^{40} T_r = 100\left[\frac{21}{2}\left\{\frac{2}{25} + 20\times\frac{1}{500}\right\}\right]$

$T_{20} + T_{21} + \ldots + T_{40}$

$= 100\left[\frac{21}{2}\times\frac{3}{25}\right] = 2\times 21\times 3 = 126$

(A) 127

(B) 126 ✓

(C) 125

(D) 124

Ans. (B)

#Q. If $a = 1 + \sum_{n=1}^{6}(-3)^{n-1} \cdot {}^{12}C_{2n-1}$, then distance of point $(12, \sqrt{3})$ from the line $ax - \sqrt{3}y + 100 = 0$ is

$\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2} \Rightarrow 2\omega = -1 + i\sqrt{3}$, $2\omega^2 = -1 - i\sqrt{3}$

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3$

1

${}^{12}C_1 - 3 \cdot {}^{12}C_3 + 3^2 \cdot {}^{12}C_5 - \ldots$

$(1+i\sqrt{3})^{12} - (1-i\sqrt{3})^{12} = 2\left[{}^{12}C_1(i\sqrt{3}) + {}^{12}C_3(i\sqrt{3})^3 + \ldots\right]$

$(-2\omega^2)^{12} - (-2\omega)^{12} = 2i\sqrt{3}\left[{}^{12}C_1 - 3 \cdot {}^{12}C_3 + \ldots\right]$

$\rightarrow 0$

(A) 55

(B) 54

(C) $\frac{109}{2}$

(D) 109

$(12, \sqrt{3})$ line $x - \sqrt{3}y + 100 = 0$

$$\text{Distance} = \frac{12 - \sqrt{3}\cdot\sqrt{3} + 100}{\sqrt{1+3}} = \frac{109}{2}.$$

Ans. (C)

#Q. If $2a_{n+2} = 5a_{n+1} - 3a_n$, where $n = 0, 1, 2, \ldots$ *If* $a_0 = 3$ *and* $a_1 = 4$, then the value of $\sum_{k=1}^{100} a_k$ is equal to

(A) $3a_{99} + 91$

(B) $3a_{100} + 91$

(C) $3a_{99} - 91$

(D) $3a_{100} - 91$

$a_0 = 3$

$a_1 = 4$

$2a_2 = 5a_1 - 3a_0$

$2a_2 = 20 - 9 = 11$

$a_2 = \frac{11}{2} = 1 + \left(\frac{9}{2^2}\right) \times 2$

$2a_3 = 5a_2 - 3a_1 = \frac{55}{2} - 3 \times 4 = \frac{31}{2}$

$a_3 = \frac{31}{4} = 1 + \frac{27}{\ }$

$a_n = 1 + 2\left(\frac{3}{2}\right)^n$

$\sum_{n=1}^{100} a_n = 100 + 2\left[\frac{3}{2} + \left(\frac{3}{2}\right)^2 + \ldots\right]$

$= 100 + 2\left[\frac{\frac{3}{2}\left(\left(\frac{3}{2}\right)^{100} - 1\right)}{\frac{1}{2}}\right]$

$= 100 + 6\left(\frac{3}{2}\right)^{100} - 6.$

$$S_{100} = 94 + 6\left(\frac{3}{2}\right)^{100}, \quad a_{100} = 1 + 2\left(\frac{3}{2}\right)^{100}$$

$$= 94 + 3\left[a_{100} - 1\right]$$

$$2\left(\frac{3}{2}\right)^{100} = a_{100} - 1$$

$$= 94 + 3a_{100} - 3$$

$$= 91 + 3a_{100}.$$

Ans. (B)

#Q. Let k_1 *and* k_2 be two randomly selected natural ~~members.~~ numbers The probability that $(i)^{k_1} + (i)^{k_2}$ is non-zero is $\left(where\ i = \sqrt{-1}\right)$

(A) $\frac{1}{6}$

(B) $\frac{1}{4}$

(C) $\frac{3}{4}$ ✓

(D) $\frac{1}{2}$

$i^{4n} = 1$ ✓

$i^{4n+1} = i$ →

$i^{4n+2} = -1$ ✓

$i^{4n+3} = -i$ →

$4n,\ 4n+1,\ 4n+2,\ 4n+3.$

Case I $k_1 = 4n,\ k_2 = 4n+2$

$P_1 = 2\left(\frac{1}{4} \times \frac{1}{4}\right) = \frac{1}{8}$

Case II $k_1 = 4n+1\quad k_2 = 4n+3$

$P_2 = 2\left(\frac{1}{4} \times \frac{1}{4}\right) = \frac{1}{8}$

$P = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}.$

Ans. (C)

#Q. In $\Delta ABC, A(4sin\theta, 4cos\theta), B(-2cos\theta, 0)$ and $C(2, 2sin\theta)$. If locus of centroid is $(3x-2)^2+(3y)^2=\alpha$ then α is

Centroid $\left(\frac{4\sin\theta-2\cos\theta+2}{3}, \frac{4\cos\theta+2\sin\theta}{3}\right)$

$3x-2=4\sin\theta-2\cos\theta$

$3y=4\cos\theta+2\sin\theta$

$(3x-2)^2+(3y)^2=4^2\sin^2\theta+4\cos^2\theta+4^2\cos^2\theta+4\sin^2\theta$

$=20$

(A) 12

(B) 16

(C) 4

(D) 20 ✓

Ans. (D)

$ae = \sqrt{a^2 - b^2} = \sqrt{5}$, $\quad b^2 = a^2(1-e^2) = a^2\left[1 - \frac{5}{9}\right] = a^2 \cdot \frac{4}{9}$

#Q. Let $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ be an ellipse and a series of ellipse are drawn that E_{i+1} has same centre eccentricity as E_i and E_{i+1}'s major axis is minor axis of E_i. If S_i be the area of E_i, then $\left(\frac{5}{\pi}\sum_{i=1}^{\infty} S_i\right)$ is equal to

A 72

B 45

C 54

D 63

$E_1 :\quad a^2 = 9,\ b^2 = 4$

$a = 3,\ b = 2,\ e = \frac{\sqrt{5}}{3}$

$E_2 :\quad a_1 = 2,\ b_1 = a_1 \cdot \frac{2}{3} = 2 \cdot \frac{2}{3}$

$3 \cdot \frac{2}{3}$

$S_1 = \pi ab = \pi(3)(2)$

$S_2 = \pi \cdot a_1 b_1 = \pi \cdot a \times \frac{2}{3} \times b \times \frac{2}{3}$

$= \pi ab \cdot \frac{4}{9}$

$S_3 = S_2 \cdot \frac{4}{9} = \pi ab \left(\frac{4}{9}\right)^2$

$$\frac{5}{\pi}\sum_{i=1}^{\infty} S_i = \frac{5}{\pi}\left[\pi ab + \pi ab\cdot\frac{4}{9} + \pi ab\cdot\left(\frac{4}{9}\right)^2 + \cdots\right]$$

$$= 5ab\left[1 + \frac{4}{9} + \left(\frac{4}{9}\right)^2 + \cdots\right]$$

$$= 5(3)(2)\cdot\left[\frac{1}{1-\frac{4}{9}}\right] = 5\times 6\cdot\frac{1}{\frac{5}{9}} = 6\times 9 = 54$$

Ans. (C)

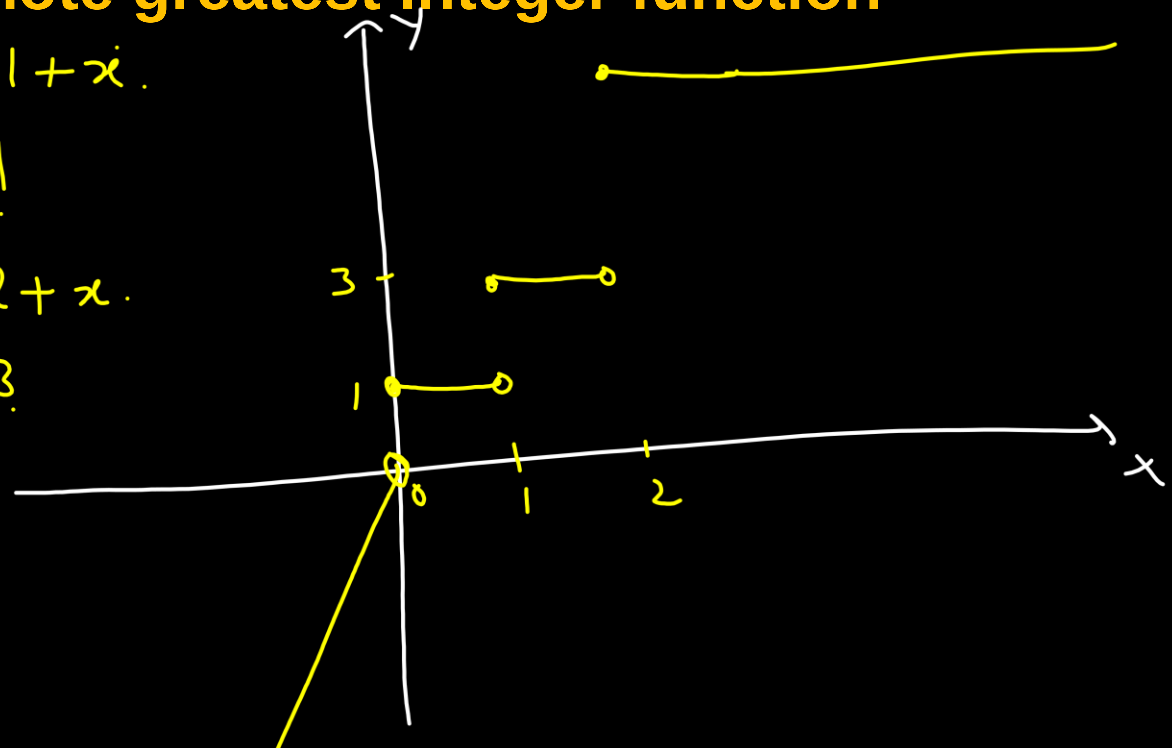
#Q. Let $f(x) = \begin{cases} 2x, & x < 0 \\ (1 + x + [x], 1 + 2[x]), & 0 \le x < 2 \\ 5, & x \ge 2 \end{cases}$. If α is the number of points of discontinuity and β is the number of points of non-differentiability, then $(\alpha + \beta)$ is equal to (where [.] denote greatest integer function

(A) 8

(B) 4

(C) 5

(D) 6



Ans. (D)

#Q. Evaluate $cos\left(sin^{-1}\left(\frac{3}{5}\right)+sin^{-1}\left(\frac{5}{13}\right)+sin^{-1}\left(\frac{33}{65}\right)\right)$ $= \cos\left(\frac{\pi}{2}\right) = 0.$

$\cos^{-1}\left(\frac{33}{65}\right)$

(A) 2

(B) 0

(C) 1

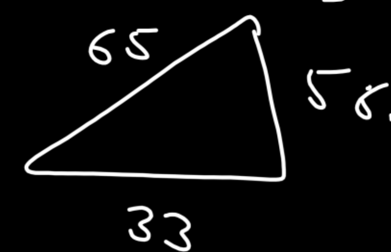
(D) $cos\,\frac{5}{13}$

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$$

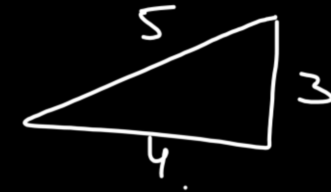
$$\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} = \sin^{-1}\left[\frac{3}{5}\cdot\sqrt{1-\frac{25}{169}} + \frac{5}{13}\sqrt{1-\frac{9}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\times\frac{12}{13} + \frac{5}{13}\times\frac{4}{5}\right]$$

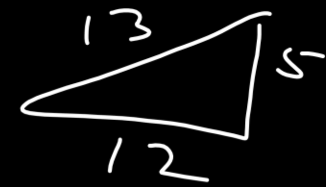
$$= \sin^{-1}\left[\frac{56}{65}\right] = \cos^{-1}\left(\frac{33}{65}\right).$$



$$\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$$



$$\sin^{-1}\frac{5}{13} = \tan^{-1}\frac{5}{12}$$



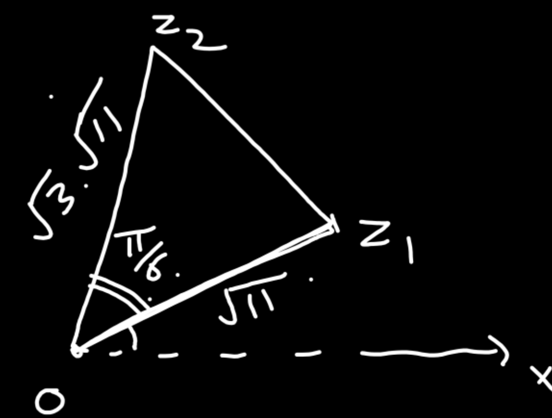
$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

Ans. (B)

#Q. $z_1 = \sqrt{3} + 2\sqrt{2}i$ & $\sqrt{3}|Z_1| = |Z_2|$ and $arg(z_2) = arg(z_1) + \frac{\pi}{6}$ then area of triangle with vertices z_1, z_2 and origin.

$|z_1| = \sqrt{3+8} = \sqrt{11}$

$|z_2| = \sqrt{3}.\sqrt{11}$



Area $= \frac{1}{2}\sqrt{11}.\sqrt{3}.\sqrt{11}\sin\frac{\pi}{6}$

$= \frac{11\sqrt{3}}{4}$

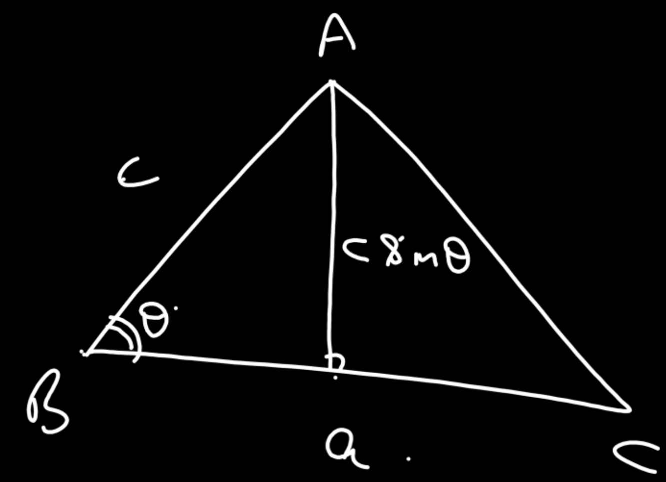
(A) $\frac{11\sqrt{3}}{4}$ ✓

(B) $\frac{11\sqrt{2}}{4}$

(C) $\frac{11\sqrt{5}}{4}$

(D) 9

$$\frac{1}{2} a c \sin\theta$$



Ans. (A)

#Q. Let R be a relation such that $R = \{(x, y): x, y \in Z \; and \; (x + y) \; is \; even\}$, then the relation R is

Reflexive: $(x+x) = 2x$ is even ✓

Symmetric: $x+y$ is even, then $(y+x)$ is even

Transitive: $x+y$ is even ✓
$y+z$ is even
$x+z$ even

(A) Equivalence relation ✓

(B) Transitive only

(C) Reflexive and transitive but not symmetric

(D) Reflexive and symmetric but not transitive

Ans. (A)

#Q. Area of region $\{(x,y): 0 \le y \le 2|x|+1, 0 \le y \le x^2+1, |x| \le 3\}$

(A) $\frac{80}{3}$

(B) $\frac{64}{3}$

(C) $\frac{32}{3}$

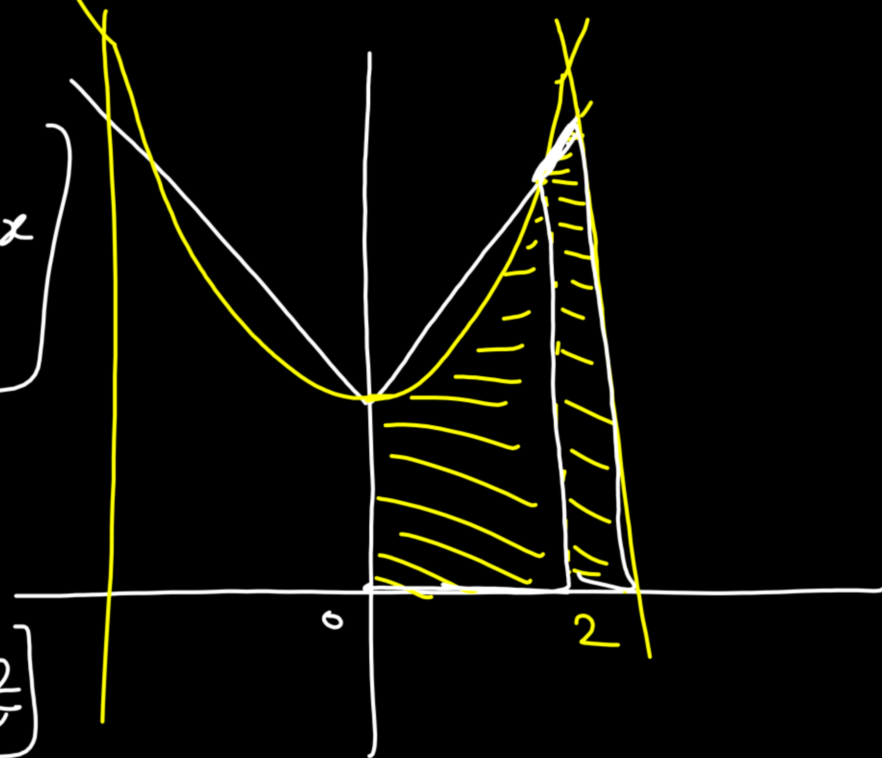
(D) $\frac{17}{3}$

$$2\left[\int_0^2 (x^2+1)\,dx + \int_2^3 (2x+1)\,dx\right]$$

$$2\left[\left(\frac{x^3}{3}+x\right)_0^2 + \left(x^2+x\right)_2^3\right]$$

$$2\left[\frac{8}{3}+2+9+3-4-2\right]$$

$$2\left[\frac{8}{3}+8\right] = 2\times\frac{32}{3}$$



Ans. (B)

JEE MAIN 2025 LIVE PAPER DISCUSSION

$|x| = x,\ x \geq 0$ & $|x| = -x,\ x < 0$.

#Q. The sum of squares of real roots of the equation: $x^2 + |2x - 3| - 4 = 0$, is

If $2x - 3 \geq 0$, $x \geq \frac{3}{2}$

$x^2 + 2x - 3 - 4 = 0$.

$x^2 + 2x - 7 = 0$

$(x+1)^2 = 8 \Rightarrow x + 1 = 2\sqrt{2}, -2\sqrt{2}$.

$x = -1 + 2\sqrt{2}$, $-1 - 2\sqrt{2}$ ✗

If $2x - 3 < 0$
$x < \frac{3}{2}$.

$x^2 - (2x - 3) - 4 = 0$

$x^2 - 2x - 1 = 0$

$(x-1)^2 = 2 \Rightarrow x = 1 + \sqrt{2}$ ✗, $1 - \sqrt{2}$

A. $6(2 + \sqrt{2})$

B. $3(2 + \sqrt{2})$

C. $3(2 - \sqrt{2})$

D. $6(2 - \sqrt{2})$ ✓

$$(-1+2\sqrt{2})^2+(1-\sqrt{2})^2$$

$$1+8-4\sqrt{2}+1+2-2\sqrt{2}$$

$$12-6\sqrt{2}$$

$$=6(2-\sqrt{2})$$

Ans. (D)

$\underline{21}$

#Q. There are 2 bad oranges mixed with 7 good oranges and 2 oranges are drawn at random. Let X be the number of bad oranges. The variance of X is

(A) $\frac{91}{206}$

(B) $\frac{63}{108}$

(C) $\frac{49}{162}$ ✓

(D) $\frac{51}{268}$

X	0	1	2
P(x)	$\frac{^7C_2}{^9C_2}$	$\frac{^2C_1 \cdot ^7C_1}{^9C_2}$	$\frac{^2C_2}{^9C_2}$
	$\frac{21}{36}$	$\frac{14}{36}$	$\frac{1}{36}$

$$\text{Variance} = \sum x_i^2 p_i - \left(\sum x_i p_i\right)^2$$

$$= 0 + \frac{14}{36} + \frac{4}{36} - \left(0 + \frac{14}{36} + \frac{2}{36}\right)^2$$

$$\frac{1}{2} - \left(\frac{16}{36}\right)^2$$

$$\frac{1}{2} - \frac{16}{81}$$

$$\frac{81-32}{162}$$

$$= \frac{49}{162}$$

Ans. (C)

#Q. If $\int_0^x t f(t) dt = x^2 f(x)$ and $f(2) = 3$, then $f(6)$ equals to

(A) 2

(B) 3

(C) 6

(D) 1

Ans. (D)

#Q. If the image of the point $\mathrm{P}(4,4,3)$ in the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{1}$ $Q(\alpha,\beta,\gamma)$, then $(\alpha+\beta+\gamma)$ is equal to

A 7

B 8

C $\frac{31}{3}$

D $\frac{11}{3}$

Ans. (C)

#Q. If $f(x)=\frac{2^x}{2^x+\sqrt{2}}, x \in R, then\ \sum_{k=1}^{81} f\left(\frac{k}{82}\right)$ is equal to

(A) 41

(B) 40.5

(C) 82

(D) $81\sqrt{2}$

Ans. (B)

#Q. Number of ways to form 5-digit numbers greater than 50000 with the use of digits 0, 1, 2, 3, 4, 5, 6, 7 such that sum of first and last digit is not more than 8, is equal to

(A) 5119

(B) 5120

(C) 4607

(D) 4068

Ans. (C)

#Q. If $\int_{-\pi/2}^{\pi/2} \frac{96\left(x^2+\cos x\right)}{1+e^x} dx = \alpha\pi^3 + \beta$ (where α, β are positive integers), then $\alpha + \beta$ equal to

A 144

B 100

C 64

D 196

Ans. (B)